Determination of the Profile of a Growing Droplet

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The profile of a growing droplet was determined with a pressure balance which was similar to that derived by Laplace for the static droplet. An additional term was added to the balance to account for the pressure on the interface due to the motion of the fluid within the droplet. The entire pressure balance was then combined with differential equations describing the geometry of the droplet. Computed profiles compared favorably with those obtained experimentally.

An additional result was the definition of a parameter, *f*, indicating the importance of internal fluid motion on the shape of the profile. A method of estimating this parameter without solving the equations was developed.

One of the most difficult quantities to measure in a liquid-liquid extraction process is the interfacial area available for mass transfer. When extraction columns are used, this area is created by droplets which form on a distributing plate, separate from the plate, and rise through a column of fluid before being collected.

A knowledge of the behavior of the droplet during the formation period is important for two reasons. First, a considerable amount of the total mass transfer may occur during this period due to the formation of a large amount of new surface area. More important, the surface area which will be available during a large portion of the rise period is determined by the size of the droplet which separates from the plate at the end of the formation period.

Very little work has been done on the development of a set of equations to describe the droplet during this period. There have been some empirical correlations based on the geometry of the droplet (*1, 2*) and a study which attempted to describe the size of the droplet at separation by using dimensional analysis (*3*), but these correlations have met with only limited success because they were restricted to particular types of profiles.

The equations which describe the profile of a static droplet have been known for some time (*4*). They should be useful since the solution for the static case must in the limit describe the profile of a slowly growing droplet. The pressure term as developed by Laplace (*5*) for the static case is concerned only with the surface curvature and the surface tension and would also apply to the surface of a growing droplet. The equations of Bashforth and Adams (*4*), which describe hanging and sessile droplets, could also be used if the growing droplet were assumed to be axially symmetric. These equations, which completely describe the static droplet, should provide a basis for describing the growing droplet if the same symmetry and interface assumptions are made. This work is concerned with the effect of fluid motion on the drop profile and the results, although tested on droplets forming on a submerged plate, should also apply to the formation of a hanging droplet (*4*).

THEORY

Any difference in the profiles of a growing and a static droplet must result from the motion of the fluid within the droplet. The normal path for an element of fluid to follow would be to flow directly from the orifice to the apex of the droplet and then parallel to the interface (Figure 1). Each segment of the interface diverts the fluid element slightly from its path, and so the fluid element causes an outward pressure on the segment. This outward pressure depends on the rate and type of flow

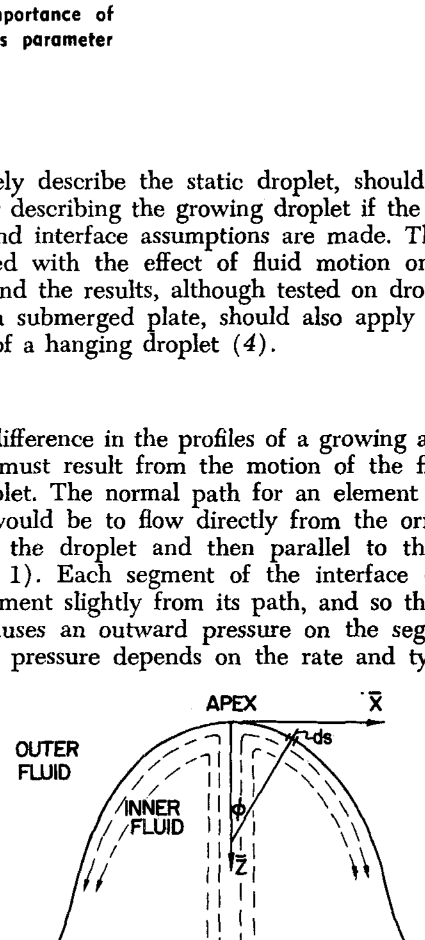

Fig. 1. Diagram indicating the assumed flow pattern and coordinate system.

within the droplet and on the nature of the interface.

In calculating the magnitude of the outward pressure, we made the following assumptions: (1) plug flow with negligible viscous dissipation due to fluid motion, (2) negligible velocity of the interface relative to the fluid elements, and (3) zero net force acting on any segment of the interface.

Under these assumptions, the impinging stream produces a normal force dF on an element ds of the stationary interface and is diverted equally in all directions in a horizontal plane tangent to the surface of the droplet (6). The outward pressure of the apex of the droplet on the interface due to the movement of the fluid is

$$\left(\frac{dF}{ds}\right)_o = \left(\frac{m^2}{\pi^2 R^4 D}\right) \tag{1}$$

This is the term which must be included in the static equations because of the motion of the fluid.

Figure 2 is a force diagram for an element of surface ds located anywhere on the interface of the growing droplet. The term (dF/ds) is again the outward pressure due to the moving fluid impinging on the element. If the forces are assumed to be balanced,

$$p_o + dg\bar{Z} + T\left(\frac{\sin\phi}{\bar{X}} + \frac{1}{\bar{\rho}}\right) = P_o + Dg\bar{Z} + \left(\frac{dF}{ds}\right) \tag{2}$$

By means of Equation (1), each of the terms in this equation may be evaluated at the origin ($\bar{X}, \bar{Z} = 0$):

$$p_o + \frac{2T}{b} = P_o + (m^2/\pi^2 R^4 D) \tag{3}$$

where b is the initial curvature as defined by Bashforth and Adams.

Equations (2) and (3) may be combined to eliminate the difference in pressure across the interface at the origin, and the result may be placed in dimensionless form by introducing the variables $\rho = \bar{\rho}/b$, $X = \bar{X}/b$, and $Z = \bar{Z}/b$.

$$\frac{1}{\rho} = 2.0 - f + \beta z - \frac{\sin\phi}{X} + \frac{b}{T}\left(\frac{dF}{ds}\right) \tag{4}$$

$$\beta = \frac{g(D-d)b^2}{T} \tag{5}$$

$$f = \frac{m^2 b}{\pi^2 T R^4 D} \tag{6}$$

The dimensionless parameter β contains the physical properties of the fluid, and f, also dimensionless, is the dynamic pressure caused by the fluid impinging on the interface at the apex of the droplet.

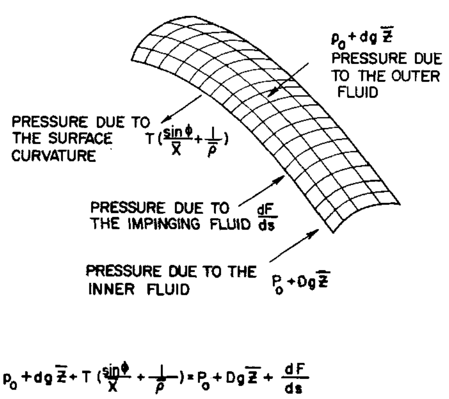

Fig. 2. Pressure balance on an element of interface.

Equation (4) has the same restriction as the equation of Laplace. The volume of the droplet must be less than the critical volume, where the critical volume is the maximum volume which the droplet may have and yet be mechanically stable.

The remaining equations and associated boundary conditions are a result of assuming an axially symmetric profile.

$$\frac{dX}{da} = \cos\phi \qquad \left(\frac{dX}{da}\right)_o = 1.0 \tag{7}$$

$$\frac{dZ}{da} = \sin\phi \qquad \left(\frac{dZ}{da}\right)_o = 0 \tag{8}$$

$$\frac{d\phi}{da} = \frac{1}{\rho} \qquad \left(\frac{d\phi}{da}\right)_o = 1.0 \tag{9}$$

Evaluating Equation (4) at the origin shows that

$$f = \frac{b}{T}\left(\frac{dF}{ds}\right)_o \tag{10}$$

and so the dynamic case reduces to the static case at the apex, and the profiles of static and growing droplets should be similar near the origin. The solution may thus be obtained by integrating the static equations for an initial small increment and then using Equations (4) to (9).

The term $(b/T)(dF/ds)$ is dependent on the rate of momentum in the stream deflected by the interface. To find the angle at which each of these streams strikes the interfacial elements, we used the average of the normal to the interface at the beginning and end of the element. For that portion of the interface which is directly above the orifice, there is an additional momentum contribution, S_r, from the fluid coming from the orifice. The rate of momentum, S, and the normal force, F, on each element is then the sum of these two contributions, as shown in Figure 3.

$$S_i = S_{i-1}\cos(\phi_i - \phi_{i-1}) - S_r \sin\phi_i \tag{11}$$

$$dF_i = S_{i-1}\sin(\phi_i - \phi_{i-1}) + S_r\cos\phi_i \tag{12}$$

where

$$S_r = \frac{m^2(X_i^2 - X_{i-1}^2)}{\pi D R^4} \tag{13}$$

$$S_1 = \frac{m^2 X_1^2}{\pi D R^4} \tag{14}$$

During the actual integration, the final X coordinate, the amount of surface area, ds, and the average normal angle, ϕ_i, for a general element are unknown. Initial estimates of these quantities were used in Equations (11) to (13) for calculation of the term $(b/T)(dF/ds)$. Equations (4) to (9) were then integrated for a preset distance along the interface. The initial estimates were compared with the computed values, and an iteration scheme was used until the assumed and computed values differed by less than 0.1%. The process was continued until the entire profile had been determined.

The magnitude of the normal force due to the deflected stream was always considerably less than that due to the stream coming directly from the orifice. When the calculated momentum component, S_i, of the deflected stream became negative, the associated force term was neglected.

Since the force due to the stream S_r becomes zero when the X coordinate exceeds the radius of the orifice, the integration proceeds more rapidly as the profile is developed. No iteration scheme is necessary if X is greater than R, and the force due to S_i may be neglected.

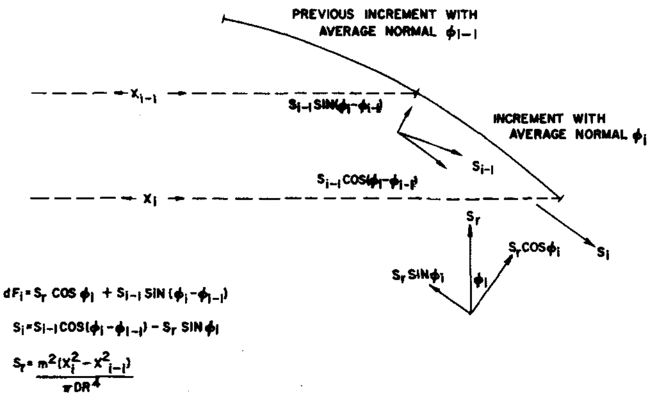

Fig. 3. Momentum balance on the *i*th increment.

EXPERIMENTAL

A Plexiglas chamber was constructed which permitted photographs to be taken of growing droplets on different plate materials. The system, a 2:1 mixture of mineral oil and Varsol (a purified fraction of C_9 and C_{10} hydrocarbons sold by Humble Oil and Refining Company) as the light phase and water as the heavy phase, was chosen for ease of handling and the size of the droplets formed under the conditions investigated. The surface tension of this system is approximately 50 dynes/cm. and the specific gravity of the light phase is about 0.84.

The growing droplets were photographed with a motorized Leica M2 camera and then enlarged so that the profiles might be measured.

Included in the parameter β is the interfacial tension between the two fluids. If mass transfer were occurring, β would vary with any changes in the concentration at the interface. If the tension were a known function of droplet volume, surface area, or position on the surface, its variation could be included in the equations. However, this would make the integration more difficult, and the determination of the required function would be difficult experimentally. Since the purpose of this study was primarily to investigate the accuracy of the dynamic terms, all the data were taken for mutually saturated fluids which permitted the tension to be considered constant during the entire growth process.

DISCUSSION OF RESULTS

When the physical properties of the fluid, the orifice radius, and the mass flow rate into the droplet are all known, the only quantity that is needed to obtain a profile by means of Equations (4) to (9) is the initial curvature b. This is the quantity which changes as the droplet grows—decreasing as the volume increases until the critical volume is reached. Each particular droplet has associated with it a particular value of b. This value could not be determined with the desired accuracy from the photograph of the droplet; therefore, the equations were integrated, several values of b being used to determine whether there was a particular value which made the computed and experimental profiles similar in shape. This was the method used to determine how well the equations described the actual profiles.

When the equations are integrated, the initial curvature influences the shape of the profile through the dimensionless groups β and f. All the possible profiles which a droplet may assume can be generated by varying these two parameters. For a particular set of fluid properties, the profiles of droplets grown on the same type of plate maintain the same general shape as the flow rate increases, because the value of b decreases to compensate for increases in the mass flow rate.

When the flow conditions are held constant, β increases to a maximum as the droplet grows to the critical volume. The magnitude of the values which β assumes during this type of growth depends upon the fluid properties and the initial curvature. The location of the range of values which β assumes during this growth may be changed by using either a different fluid system or a different plate material.

There are, in general, only two regions which produce droplets with distinctly different profiles. These profiles correspond to droplets which either spread out on the plate or do not wet the plate at all and have bases which are restricted to the orifice edge. Mathematically, one profile is normally single valued, and the other is a double-valued function at the critical volume.

As mentioned previously, these two types of droplets may be obtained experimentally by changing either the fluid system or the plate material. Since any change in the fluid system would introduce possible errors due to the measurement of fluid properties, only the plate material was changed. For the system used in this investigation a change from a Teflon to a stainless steel plate resulted in the desired change in the profile.

When Teflon plates were used, the droplet spread out on the plate, and it was difficult to determine experimentally whether the base of the droplet was symmetric with respect to the orifice. If the base was not symmetric, the interface would no longer be a surface of revolution, and one of the assumptions would be violated. The only way to check this was a careful examination of the photographs. In almost all droplets grown on this type of plate there would be some error due to this difficulty. Several different comparisons of the profiles of droplets grown on Teflon plates are plotted in Figures 4 through 6.

The effect of the dynamic terms on the solution can be seen in Figure 5, where the profile that results when these terms are neglected is also plotted. Another measure of the importance of the dynamic terms is the dynamic pressure f. An examination of Equation (4) indicates that if f is small relative to 2.0, then the dynamic terms can probably be neglected. For the flow conditions of the droplet in Figure 5, f was approximately 0.96.

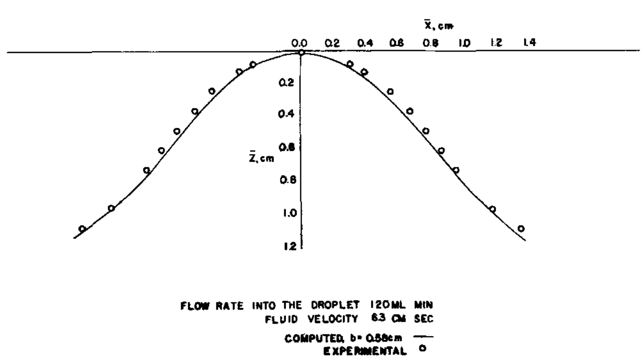

Fig. 4. Comparison of profiles—Teflon plate, 0.25-in.diameter orifice, $f = 0.39$.

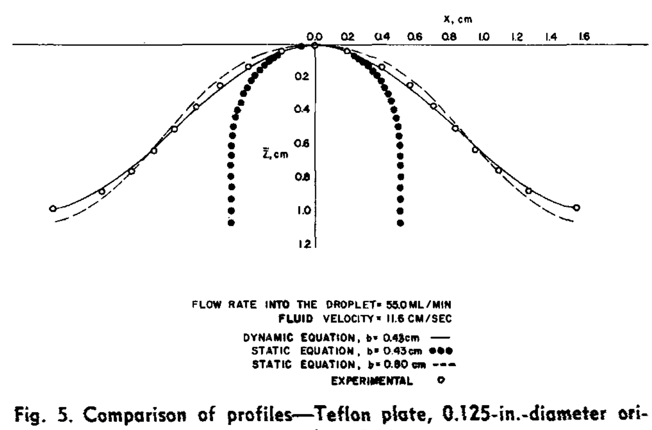

Fig. 5. Comparison of profiles—Teflon plate, 0.125-in.-diameter orifice, $f = 0.96$.

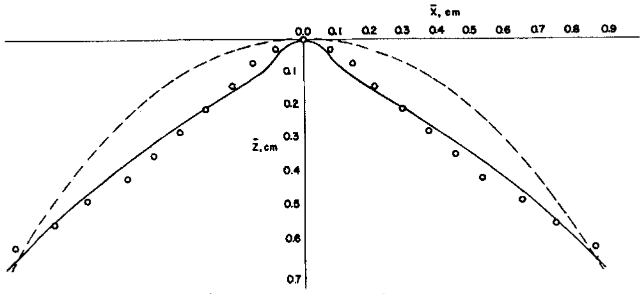

Fig. 6. Comparison of profiles—Teflon plate, 0.0625-in.-diameter orifice, $f = 1.82$.

Also plotted in Figure 5 as well as in Figure 6 is the profile corresponding to the b value which agreed with the experimental data if the static equation was used. As expected, the two families of curves for the static and dynamic cases are similar, but Figure 6 shows that the static equation is inadequate in this case.

Since the surface area and the volume of static droplets have been tabulated, it would be desirable to use the static equation to obtain an initial estimate of the profile. Practically, the static equation may be used as long as the value of the dimensionless group f is small compared to 2.0. In order to calculate f, one needs a value of the initial curvature b. This value may be obtained by estimating β and then calculating b from Equation (5). β is almost never less than -4.0 and may be assumed to be -1.0 if no other information is available. This should give a value of b which will probably be conservative for most sessile droplets with a significant volume.

When a stainless steel plate was used, the droplet did not wet the plate at all. Since the base of the droplet is determined by the edge of the orifice in this case, the interface will be a surface of revolution as long as the plate is perfectly level. The computed and experimental profiles of a droplet growing on this type of plate are compared in Figure 7. The equations appear to provide a better description of droplets growing on this type of plate than on Teflon, probably because these droplets more closely approximate a surface of revolution.

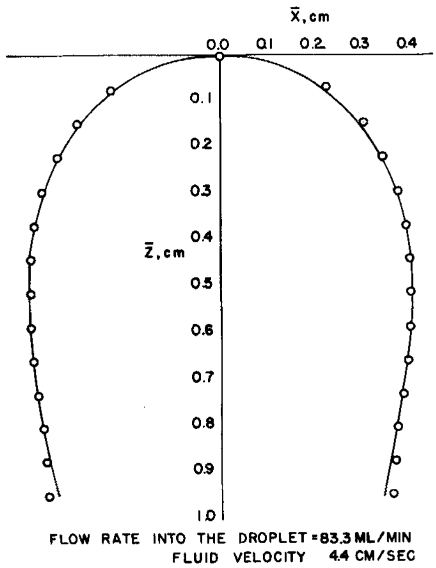

Fig. 7. Comparison of profiles—stainless steel plate, 0.25-in.-diameter orifice, $f = 0.12$.

CONCLUSIONS

The development appears to be a reasonable representation of the actual profile of a growing droplet for the conditions investigated. Unfortunately, it applies only as long as the volume of the droplet is less than the critical volume. At this point, a description of a growing droplet whose volume exceeds the critical volume is needed. The results of this investigation should be useful in such a study, since the influence of the fluid motion should be the same in either case. It should be necessary only to investigate the separation of a droplet whose volume exceeds the critical volume at no flow conditions. The terms regarding the fluid motion could then be included in the equations. If this additional development were available, the interfacial area as a function of flow rate, orifice size, and fluid properties could then be calculated.

On the basis of this work, an estimate of the importance of the dynamic terms can be made before any integration by estimating f from the parameter β. When f is small relative to 2.0, the tabulated data for static droplets may be used in determining the profile of a growing droplet without appreciable error. When f is large, the equations developed here should be used.

NOTATION

a = arc length along the profile
b = radius of curvature at the origin, cm.
D = density of the fluid composing the droplet, g./cc.
d = density of the external fluid, g./cc.
F = normal force due to the impinging fluid, dynes
f = dimensionless group
g = gravitational constant, 980 cm./sec.^{2}
m = mass flow rate, g./sec.
P = pressure due to the internal fluid., dynes/sq.cm.
p = pressure due to the external fluid, dynes/sq.cm.
R = orifice radius, cm.
S = momentum rate, dynes
s = surface area, sq.cm.
T = surface tension, dynes/cm.
X = interface coordinate, $X = \overline{X}/b$
Z = interface coordinate, $Z = \overline{Z}/b$
β = dimensionless group
ϕ = normal angle, radians
ρ = radius of curvature of the droplet, $\rho = \overline{\rho}/b$

Superscripts

$-$ = dimensioned variable, cm.

Subscripts

o = origin
i = increment number
r = orifice

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Manuscript received March 31, 1967; revision received July 3, 1967; paper accepted July 5, 1967. Paper presented at AIChE Salt Lake City meeting.